Elasticities – a Theoretical Introduction

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Abstract

The purpose of this appendix is to give a basic theoretical description of the concept of elasticity. The concept comes from economic theory and is linked to the description of demand adjustments in a market – consumer as well as producer demand. The elasticity can be defined as a correlation between two variables, e.g. price and demand. When price increases demand typically decreases and the size of the decrease is determined by the elasticity – in this context called the elasticity of demand. In empirical oriented economic research the concept of elasticity is first and foremost linked to the so-called equilibrium models describing supply, demand and pricing on a market, e.g. the electricity market. In such models the elasticities play an important role because they determine the size of the demand adjustments that come about as a result of price changes on the market.

Contents

Figures

Tables

1 Introduction

In order to get a measure of how demand for a good reacts to price changes one could study the slope of the demand curve. However, this method might cause problems because it depends on the unit in which the good is measured (e.g. kWh as regards electricity). In order to avoid unit problems the concept of elasticity has been developed. Here the percentage change in demand as a result of a percentage change in price is measured. In this way you get a concept independent of units.

The elasticities are used in order to examine how sensitive the demand for a good is to changes in the price of the good itself, to changes in the price of related goods, and to changes in income. As the demand for a good depends on more factors than the price of the good itself we have to introduce various types of elasticities. Section 2 will give a general introduction of the following concepts of elasticity: own price elasticity, income elasticity, cross-price elasticity and elasticity of substitution. In Section 3 the elasticities will be examined from the choice of different types of function. Here we attach weight to the functions Cobb-Douglas and CES. Besides, the Leontief function, the Stone Greary function and the CET function will be mentioned briefly. At the end of the section a short discussion will take place of whether to model the demand side with one representative consumer or whether to use more consumers. There will also be a discussion of different welfare measures. Section 4 briefly points out which considerations a company faces when the profit is going to be maximized. Section 5 gives an account of the choice of functions used in the ELEPHANT model.

2 Types of Demand Elasticities

The demand side in an equilibrium model is characterised by the consumers maximizing their utility (*U*). The utility derives from consumption (*C*) of the goods that are included in the economy. Thus the general term of the utility function could be written as $U=U(C_1,C_2,...,C_i)$. This function should be maximized provided that the consumer can afford the goods. This restriction is called the budget constraint and could be written as $\sum_{i=1}^{n} p_i C_i \leq M$

where M is the consumer income and p is the price vector.

In optimum the consumer will spend the whole income on goods unless he benefits from saving up. If the consumer does not have saving opportunities the restrictions would be indicated by "=" and not "≤" when he maximizes his utility.

2.1 Own-price elasticity

Own-price elasticity – or simply price elasticity as the concept is called, too – shows the percentage rise in the demand at a percentage rise in the price of the good itself.

As the demand curves generally have a negative slope the own-price elasticity turns negative too, which corresponds to a decline in the demand when the price increases. The negative slope of the demand curve is caused by the fact that the demand is always given as a function of the price and the curve is depicted in a coordinate system with the price on the y-axis and the quantity on the x-axis. Yet, often the own-price elasticity is determined numerically in order not to question whether the demand for a good with an own-price elasticity of -3 is more or less elastic than the demand for a good with an elasticity of -2. Formalized, the own-price elasticity is given as:

$$\varepsilon_p = \frac{\partial C_i / C_i}{\partial p_i / p_i} = \frac{\partial C_i}{\partial p_i} \frac{p_i}{C_i}$$

where C_i defines the demand for good *i* and p_i defines the price of good *i*. If $|\varepsilon_p| = 1$ the demand is defined as being unit elastic while the demand is defined as being elastic if $|\varepsilon_p| > 1$ and inelastic if $|\varepsilon_p| < 1$. If for instance $|\varepsilon_p| = 1$ it means that a price increase of 1% will cause a reduction in the demand for a good of 1%. On the opposite, a price decrease of 1% will cause an increase in demand of 1%. The expense of a good will, however, remains the same when the demand is unit elastic. If the demand is inelastic a price increase means that the decrease in the purchased quantity will be relatively smaller than the increase in price. So the consumer's total expense for the good in question increases. The opposite is the case at a price increase of a good where the demand is elastic (Fog, 1992).

Own-price elasticity changes over time. This might be due to timelag in information about price levels and lack of information about substitution possibilities. This adjustment process makes the long-term elasticities bigger than the short-term elasticities. A survey in the USA about elasticities of different goods shows that the short-term elasticity of electricity in households is 0.13 while the long-term elasticity is 1.89 (Salvatore, 1993). In the Danish macroeconomic model ADAM the long-term elasticity for electricity in the households has been estimated at 1.35 (Møller Andersen, 1997).

2.2 Income elasticity

Income elasticity shows the percentage increase in the demand for a given good as a result of a percentage increase in income.

Formalized, the income elasticity is given as:

$$\varepsilon_I = \frac{\partial C_i / C_i}{\partial M / M} = \frac{\partial C_i}{\partial M} \frac{M}{C_i}$$

M indicates the income.

Generally, the income elasticity for necessaries is smaller than for luxury goods. So a reduction in income will not reduce the consumption of for instance electricity just as much as for instance the consumption of holiday trips. It is also the case that the income elasticity of the good decreases when the income increases. In the short run, the income elasticity of the fixed part of the household budget is equal to 0, while variations in consumption of the goods which are part of the variable costs of the budget increase. In the long run, this would not be the case because several items on the budget could be varied (Fog, 1992). In surveys from the USA the income elasticity of electricity in households was estimated at 1.94 (Salvatore, 1993).

2.3 Cross-price elasticity

Cross-price elasticity shows the percentage increase in demand for good *i* as a result of a percentage increase in the price of good *j*.

The mathematical definition of cross-price elasticity is given as

$$\varepsilon_{i,j} = \frac{\partial C_i / C_i}{\partial p_j / p_j} = \frac{\partial C_i}{\partial p_j} \frac{p_j}{C_i}$$

Cross-price elasticity for a good having a close substitution or complementarity would numerically be relatively big. If there is a close substitution the cross-price elasticities will be positive as a price increase of good *i* will make the consumers substitute towards demanding good *j*. If *i* and *j* are complementary goods the cross-price elasticity will be negative. A reduction in the demand for good *i* as a result of a price increase of the goodwill also leads to a decreasing demand for good *j*. For goods that are neither close substitutes nor complementary goods the cross-price elasticity will be insignificant. Generally, electricity has no close substitutes in the short run. However, there could be some substitution between the consumption of electricity and gas for cookers in the long run. Electric heating can also be substituted by e.g. natural gas. According to a survey in the USA the cross-price elasticity between electricity and natural gas in the households is 0.20. That is, if the price of electricity increases by 1% the demand for natural gas will increase by 0.20% (Salvatore, 1993).

2.4 Elasticity of substitution

Elasticity of substitution measures the percentage change in the relative consumption of two goods as a consequence of a change in the relative prices of the goods. Thus, an increase of 1% in the relation between the two factor prices will push the relation between the factors Φ % in the direction of the factor which has become relatively cheaper.

Mathematically, the elasticities of substitution are defined in the following way:

$$\sigma_{i,j} = \frac{-\partial\left(\frac{C_i}{C_j}\right)}{\partial\left(MRS_{ij}\right)} \frac{\left(MRS_{ij}\right)}{\left(\frac{C_i}{C_j}\right)}$$

MRS indicates the marginal substitution relationship between good *i* and good *j*. This corresponds to the slope of the indifference curve. In optimum, it applies that the slope of the indifference curve is equal to the price relation among the goods. This means that $MRS_{ij} = \frac{-p_i}{p_j}$

The bigger the elasticity of substitution between good *i* and good *j* the more substitutional are the goods. Opposite, if the elasticity of substitution approaches zero the good *i* and *j* will be complementary goods (i.e. two kinds of goods typically used together). The size of the elasticity of substitution will influence the slope of the indifference curve[1] (when it is a question of utility function) and the isoquant curve[2] (when it is a question of production function). Thomsen (1999) shows the slope of the isoquant curves for different values of elasticities of substitution. From this it appears that the curve converges towards a straight line when the elasticity of substitution approaches infinity, while the curve converges towards making a 90 degree kink when the elasticity of substitution approaches zero. The economic interpretation of these figures is that if the elasticity of substitution is modest (less than 1) a large price increase on one of the production factors would not be able to completely squeeze out the factor of the production process. Consequently, all factors will have an asymptotic minimum level. Reverse, the consumption of a production factor will be considerably reduced when increasing the price of the factor if the elasticity of substitution is big.

High elasticity of substitution Low elasticity of substitution

Figure 2.1. High and low elasticity of substitution

Elasticity of substitution and the cross-price elasticity are closely related as they both express something about how the demand for a good changes due to a price change of another good. The difference of the two concepts is that the cross-price elasticity does not take into account the price sensitivity of the good which price has been changed, but the elasticity of substitution does. That is, even if two cross-price elasticities are identical this does not mean that the corresponding elasticities of substitution are identical too. This is due to the fact that a decline in the demand for goods with increasing prices could be different even though the increase in the demand for a third good as a consequence of substitution could easily be the same. This could be clarified by an example. Let the cross-price elasticity between good 1 and good 3 be the same as between good 2 and good 3. This means that a price increase of good 1 or good 2 has the same influence on the consumption of good 3. If the price sensitivity for good 1 is bigger than for good 2 this means that the consumption of good 1 will decrease more than the consumption of good 2 when the price of the two goods

[1] The indifference curve indicates the quantity combination between two goods giving the consumer the same utility.

[2] The isoquant curve indicates how a given quantity could be produced with different combinations of factors of production, e.g. labour and capital.

increases. So the elasticity of substitution becomes bigger between good 1 and good 3 than between good 2 and good 3.

If you choose a functional type with a constant elasticity of substitution you gain the advantage that the percentage change in the relative consumption of the two goods is independent of the level of prices and consumption. Furthermore, the elasticity of substitution will also be independent of the price development of other goods. If the function does not have a constant elasticity of substitution you have to consider where to measure the elasticity on the demand curve.

2.5 Compensated or Non-compensated Elasticities

The general equilibrium theory has two main approaches of building a CGE model (CGE: Computed General Equilibrium). One approach is the indirect utility function approach that makes use of either the Marshallian utility functions or the indirect utility functions. Regarding the Marshallian utility function the utility is directly calculated as a function of the purchased goods or the consumption. Regarding the indirect utility function the utility is calculated as a function of prices and income. The other approach is the expenditure function approach. Here the Hicksian demand functions are used. They depend on prices and utility level. The fundamental difference between the Hicksian demand function and the general or Marshallian demand function is that when you consider the change in the Hicksian demand at a price increase on a good the consumer should have the same utility level before and after the price increase. Therefore, we assume that the consumer gets compensated for the price increase through a rise of income. Consequently, the income effect is disregarded so that only the substitution effect is left. The opposite applies to the Marshallian demand, i.e. the income is constant while the utility level might change (Pedersen, 1998).

The great advantage of the »family« of CES functions (see below) is that it is very easy to rephrase the utility function to the indirect utility function or the Hicksian utility function. When using the Hicksian demand functions it is necessary to use the compensated elasticities. These elasticities are defined in the same way as the non-compensated or general elasticities except that they are calculated on the basis of the Hicksian demand function and not on the Marshallian demand function which is used for the non-compensated elasticities.

3 Functional Types

When choosing which functional types to use in a general equilibrium model several aspects have to be considered. It is desirable to choose a functional type which is as flexible as possible regarding the elasticity characteristics. The disadvantage of choosing a functional type being more restrictive is that you have to specify more exogenous parameters. This might cause problems as it is not always easy to find realistic data determining the parameter values. So an advanced model could end being bad at describing the economy because the uncertainty in the exogenously defined parameters is too big. This section will give an account of the characteristics of Cobb-Douglas and CES functions was the most used functions in CGE models.

There is a difference between how the elasticities are used from model to model. In the empirical models you will often try to determine the characteristics of the utility function from the characteristics of the elasticities. In the theoretical models you often do it in the opposite direction and look at which elasticity characteristics that arise as a result of choosing specific functional types. In the paper we attach importance to the theoretical approach. The mathematical calculations will not be accounted for here. They can be found in Pedersen (1998).

When choosing functional type for the general equilibrium model it is important that the following conditions are fulfilled: The function must be non-negative, continuous and homogeneous of degree 0 in the prices and furthermore, Walras' law must be met. In short, this law says that the demand value must be equal to the initial resources of the economy. These restrictions are used in order to guarantee an equilibrium model and to see that the solution is included in the inner and not in a corner of the solution area which is the condition of being sure that the solution is optimal (Shoven, 1992).

3.1 The Cobb-Douglas Function

The biggest advantage of choosing the Cobb-Douglas function is that it is much easier to work with and make calculations on than the other functional forms that fulfil the above-mentioned criteria. This is due to the fact that there is no need for determining any parameters exogenous (Petersen, 1998). On the other hand, this has a great deal of weaknesses when it comes to flexibility and elasticities. Table 3.1 provides a statement of the characteristics of the Cobb-Douglas function.

Table 3.1 The Cobb-Douglas Function

Matematical form:	$A\prod_{i=1}^{n} C_i^{\alpha_i} = A\left(C_1^{\alpha_1} C_2^{\alpha_2} \cdots C_n^{\alpha_n}\right)$
Own-price elasticity	-1
Income elasticity	1
Cross-price elasticity	0
Elasticity of substitution	1
Compensated own-price elasticity	$\alpha_i - 1$
Compensated cross-price elacticity	α_j

Note: A>0 is a constant $\alpha_i > 0$, $\sum_i \alpha_i = 1$

The compensated own-price elasticity is numerically smaller than the non-compensated (general own-price elasticity). The cause for this is that the non-compensated elasticity is found by looking at the percentage change in the price for a maintained income level, whereas the compensated own-price elasticity is calculated by maintaining the utility level. The difference between the two elasticities corresponds exactly to the total proportion of budget a_i which the consumer uses on good *i*. That is, the bigger the proportion of the budget being used on good *i*, the more the consumer is affected by a price increase on good *i*. The same argumentation could also explain why the compensated cross-price elasticity is numerically bigger than the non-compensated one.

3.2 Constant Elasticity of Substitution (the CES Function)

This form of function avoids some of the restrictive limitations the Cobb-Douglas function is subordinated to. Most important is that the prices of all goods influence the demand for each single good. In the case of the Cobb-Douglas function only the price on the good itself influenced the demand. In addition, the own-price elasticity could vary freely if CES functions are used in preference to Cobb-Douglas, where the own-price elasticity is set at 1. This greater liberty is achieved just by introducing one exogenous parameter, namely the elasticity of substitution. See Table 3.2 for an overview of the characteristic of the CES function.

Table 3.2. The CES function

Matematical form:	$\left(\sum_{i=1}^{n} \beta_i C_i^{\frac{E-1}{E}}\right)^{\frac{E}{(E-1)}}$
Own-price elasticity	$-E+(E-1)e_i$
Income elasticity	1
Cross-price elasticity	$(E-1)e_j$
Elasticity of substitution	E
Compensated own-price elasticity	$-E(1-e_i)$
Compensated cross-price elasticity	Ee_j

Note: β and E are constants. e_i and e_j are the proportion of expenses for good *i* and for good *j* respectively

The interpretation of the term own-price elasticity is that it consists of the sum of the direct impact on the demand from a price increase on good *i* and the indirect effect through a price increase in the total price index. The indirect effect is equal to the share of expenses multiplied by the elasticity regarding the price index. The cross-price elasticity is equal to the elasticity in the demand for good *i* at an increase of the price index multiplied by the share of expenses for good *j*. Comparing the elasticity terms in Table 3.1 and Table 3.2 shows the evident connection that the Cobb-Douglas function is just a special case of the CES function where the elasticity of substitution *E* is set at 1.

3.3 The Nested CES functions

The problem with the CES function is that it implicates that the elasticity of substitution is identical for all pair of goods. Furthermore, its weakness is that the income elasticity cannot be freely defined, but is set at 1.

In order to deal with the first problem you could choose to arrange the CES function in nest. This means that the arguments of the function are split up into pairs. In practice this is done by dividing the function into sub-functions all being CES functions. These sub-functions could also form pairs with each other. In this way different elasticities of substitution could exist between the aggregates that are represented by the sub-functions.

One example of a nested CES function could be a production function in which capital, labour and energy are included. Here capital and labour could be matched giving them a elasticity of substitution. Then energy and the aggregate of capital-labour could be matched and give this pair another elasticity of substitution. Section 5 shows the net structure for the households' utility function and production function in the ELEPHANT model. The nested CES functions have thus a very wide flexibility regarding elasticities of substitution. The only problem is, however, to decide which inputs to be matched with each other. In the above-mentioned example energy and capital might have been put together and subsequently matching the aggregate with labour. How to choose to match the pairs depends on the empirical analyses. The flexibility in the elasticities of substitution in the nested CES function has made this the most used functional form in general equilibrium models.

The nested CES function has – just like the non-nested one – the restriction that the income elasticity is set at 1.

3.4 Other Functions

From the above tables we saw that the Cobb-Douglas function is a special case of the CES function where the elasticity of substitution is set at 1. The CES "family" also includes other functional forms. One of the most common ones is the Leontief function which corresponds to the special case where the elasticity of substitution is set at 0. Therefore, there is no possibility of substituting among the goods when this functional form is used. The last function in the CES "family" to be mentioned here is the CET function (Constant Elasticity of Transformation). The only different of this and the CES function is that the CET function represents the transformation aspect instead of the substitution aspect. If you want to use a type of function which is not bound up with an income elasticity of 1 you could choose a Stone-Geary function.[3] This type of function could be modelled either on the background of a Cobb-Douglas function

$$\left(\prod_{i=1}^{n}(C_i - K_i)^{\alpha_i}\right) \text{ or as a CES function } \left(\left[\sum_{i=1}^{n}\beta_i(C_i - K_i)^{\frac{E-1}{E}}\right]^{\frac{E}{(E-1)}}\right)$$

This is done merely by subtracting a constant from consumption C_i in the two functions. Constant k_i indicates the minimum consumption for each individual good. Contrary to the CES function it has the characteristic that the income is not multiplicatively but additively included in the utility function. In this way you can avoid that the expansion path does not become a straight line through origo which is the case with the CES function (Petersen, 1998). The expansion path shows the optimal consumption between two goods at different income levels and at fixed prices. The problem with this type of function is that the minimum consumption for all goods ought to be specified (unless it is set at 0), which could produce quite a lot of parameters which should be calibrated exogenously.

[3]The Stone-Geary function is also called LES function (linear expenditure system).

3.5 One Representative Consumer Versus More Consumers

In the general equilibrium models it is traditional to model the demand side by use of one representative consumer so that the consumer's utility is maximized. This tradition dates from the time where the calculation power of the computers was not as big as it is today. Nevertheless, you still see several examples of choosing one representative consumer. This is primarily due to the fact that in practice it is difficult to distinguish between different consumers' preferences. We have chosen to use one representative consumer in the ELEPHANT model as well because we assume that the Nordic consumers have identical preferences.

Even though today we have the technical aids for making models with many consumers this is not always desirable. This is due to the fact that the more consumers being modelled in the economy the more complicated it gets to interpret the results of the model. On the other hand, it is also the goal of an equilibrium model to describe the economy as precise as possible. So if you can get good information on the preferences of different population groups it would be good to include more consumer groups in the description of the demand side.

3.6 Welfare-economic Measures

When measuring whether a change from one scenario to another in an equilibrium model leads to an improvement or a worsening of the consumers' situation the point of departure could be different welfare measures. Some of the most used approaches to the problem is to look at either the consumer and producer surplus or at the equivalent variation. The consumer surplus is measured by calculating the area between the demand curve (corresponding to the consumer's willingness to pay) and the supply curve left of the equilibrium point. This method has for example been used in the Norwegian electricity market model, Normod. However, more problems arise when using this method. First, the method is built on the assumption of a constant marginal utility of income. Secondly, the consumer surplus can only be measured if the prices change one at a time (Frandsen, 1995). The latter is of course only a problem when there is more than one endogenous price in the model. In order to overcome these problems one could choose to use the equivalent variation. It measures the difference between consumer expenses before and after the change. In models with more consumers the equivalent variation is very useful because it measures in money and can therefore be added across the consumers (Petersen, 1998). Instead of the equivalent variation the compensated variation could also be used. The difference of the two welfare measures is that the equivalent variation takes its point of departure in the old price vector and thus the initial equilibrium while the compensated uses the final equilibrium and thus the new price vector. Therefore, the compensated variation measures how much money the consumers are going to get (or pay) in order to be compensated for the utility change that has happened in the new scenario (Shoven, 1992).

4 Production Side

In the same way as the consumers maximize their utility on the demand side it is assumed that the companies maximize their profit on the production side. In economic theory a company's production is determined by a production function specifying which inputs the company uses in order to produce its good. In general, the production function is written as $Q = f(L, K)$, where Q denotes the produced amount and L and K are input factors (usually labour and capital are used) used for the production of Q. The company therefore has to maximize the profit $\pi = P \cdot Q - C$, where P is the price the company gets for its goods and C is the company's expenses. The profit has to be maximized on the restriction that the company can afford to pay for the input factors. This restriction could be written as $P \cdot Q \leq W \cdot L + R \cdot K$, where W is the labour costs and R corresponds to payment of capital. In optimum the company will produce until the marginal revenue of input factors equals the price of the payment of these. Just as on the demand side the production side can also use different types of function like e.g. Cobb-Douglas, CES and Stone-Geary functions. The terms of elasticity on the production side are also like the ones used on the demand side.

5 Functions in the Elephant Model

This sections will exemplify the above-mentioned theory by looking at the functions used in the ELEPHANT model (Hauch, 1999). It is a model developed in order to analyse the consequences of the liberalization of the Nordic electricity market. The model is a partial equilibrium model consisting of a combination of a top-down and a bottom-up model.

In the ELEPHANT model elasticities of substitution and income elasticities are used. The elasticities of substitution is used in a nest of functions in the households' utility function and in the production function, respectively. The income elasticity in this model is used to determine the contingent parameter which states the consumption of the different goods in the basis year.

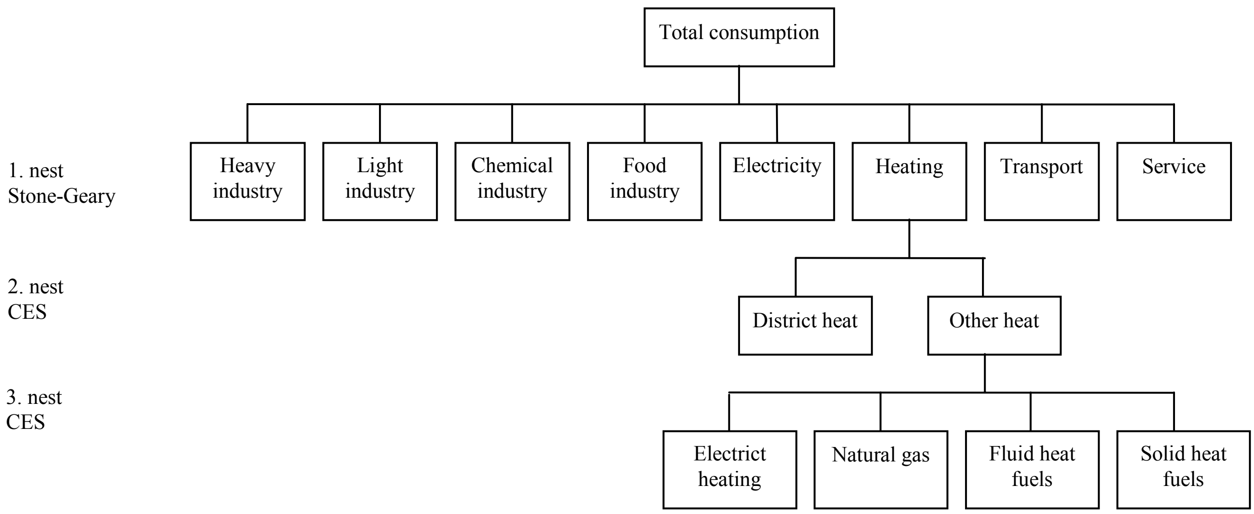

Figure 5.1. The utility function of the households

Figure 5.1 shows the nest structure of the households. It is a mixture of different functional types.

In the top nest the total budget is divided into 8 commodity groups. Here a Stone-Geary utility function is used. This type of function is additive, which gives some strong a priori restrictions on the substitution effects. These restrictions can only be justified when it is a question of few, but broad commodity groups that individually satisfy different needs. The reason why this type of function has been chosen in the model is that it only needs a limited number of parameters compared to other additive functions (Hauch, 1999). In the next nest heating is divided into district heating and other forms of heating. In this nest a CES function is used. Other forms of heating consist of an aggregate of four different types of heating. In the third nest they are also described by a CES function. This nest structure means that the elasticities of substitution between district heating and other forms of heating do not necessarily have to be the same as the substitution elasticities between for instance electric heating and natural gas.

The production function in the ELEPHANT model is like the utility function described by a nest of functions. This structure is shown in Figure 5.2.

In the top nest the companies can choose between two inputs: Energy and BNP. Here the BNP is interpreted as all other production factors but energy, like e.g. labour, capital and land. The connection between the two inputs is modelled as a CES function. In the other nest the aggregate energy is split up into electricity and other forms of energy. Again, a CES functions has been chosen. The third nest describes the connection between natural gas and the remaining energy inputs. This nest has also been described by a CES function. Finally, in the last nest a Leontief function is used to describe the connection between the remaining energy inputs. The Leontief function has been chosen because by and large no substitution possibilities exist between the individual types of energy. This means that a price increase on one type of energy does not affect the demand for the other types of energy.

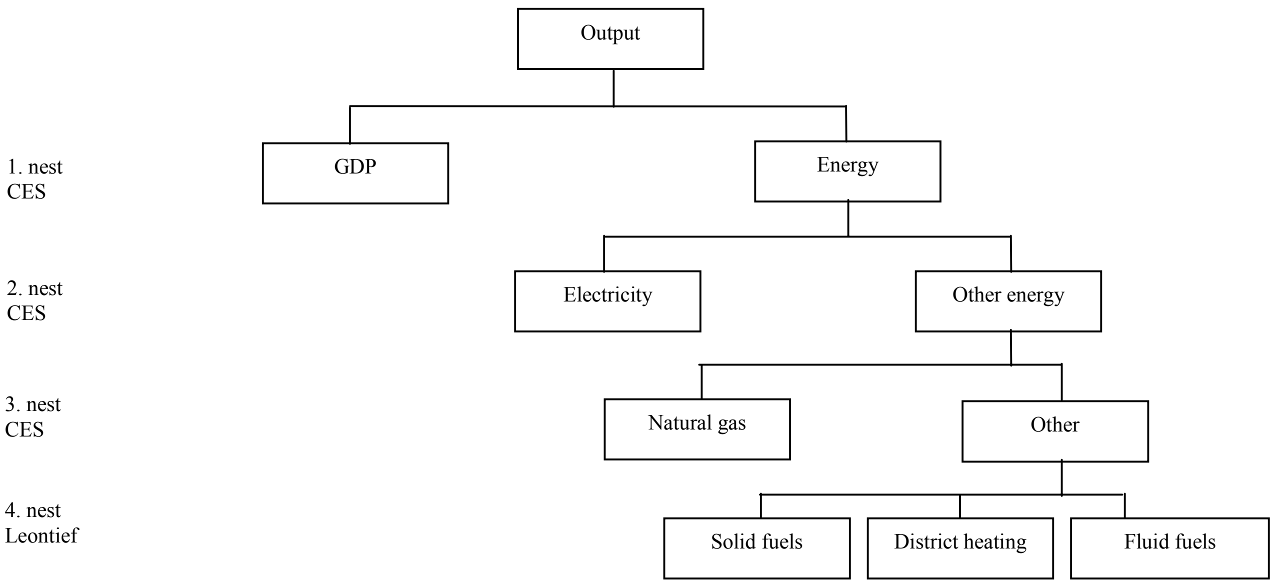

Figure 5.2. The Nest Structure of the Production Function

The following tables show the size of the elasticities included in the model. It is worth noticing that no elasticities of substitution are included in the households 1. nest as it is modelled by a Stone-Geary function. Likewise, the elasticity of substitution in 4. nest in the production function is zero, because this nest has been described by a Leontief function.

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